Effective Description of Brane Terms in Extra Dimensions

José Santiago

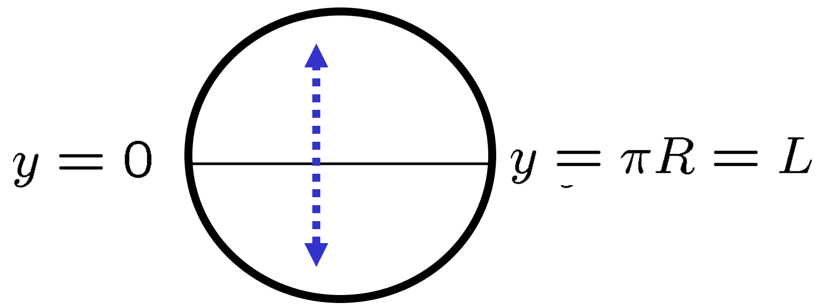
Theory Group (FNAL)

With F. Aguila (Granada) and M. Pérez-Victoria (CERN), '03-'06

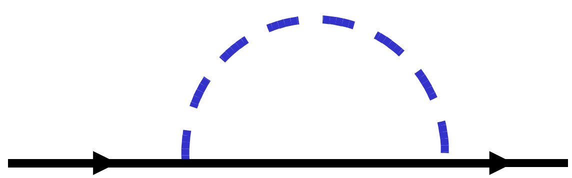
Why Brane Terms? (I)

- Translation invariance broken by branes

$$S^1/Z_2$$



- Localized divergences at one loop cancelled by counterterms:
  BKT radiatively generated even if absent at tree level



Georgi, Grant, Hailu
PLB'01

$$\delta\mathcal{L} \propto \ln\left(\frac{\mu}{M}\right)\left[\delta(y) + \delta(y - L)\right]\left[\bar{\psi}^{+} i \not{\partial}\psi^{+} + \bar{\psi}^{+}\partial_y\psi^{-} + (\partial_y\bar{\psi}^{-})\psi^{+}\right]$$

Why Brane Terms? (II)

- BKT can be very important for phenomenology

Dvali, Gabadadze, Porrati, PLB'00

$$S = M^3 \left[\int d^5x \sqrt{G} \mathcal{R}_{(5)} + r_c \int d^4x \sqrt{g} R \right]$$

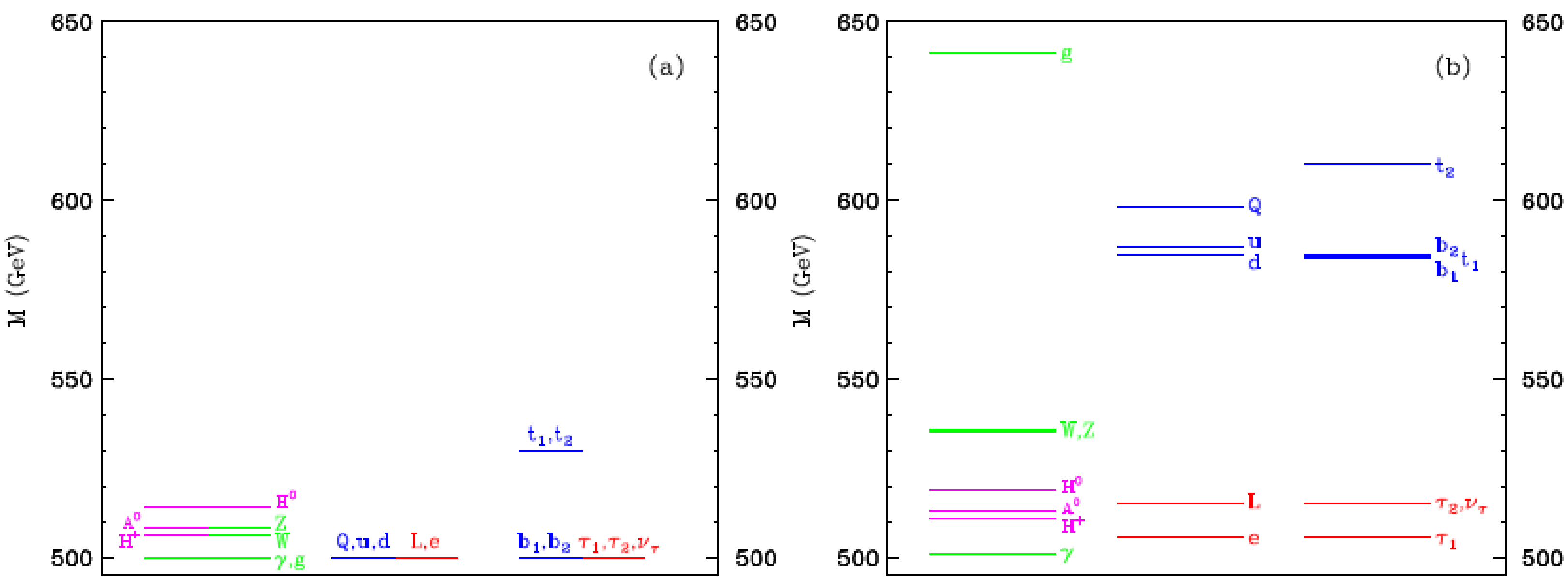
  - 4D gravity at low distances
  - 5D gravity at distances larger than r_c

- We will focus on field theory examples

Why Brane Terms? (II)

- BKT can be very important for phenomenology

Cheng, Matchev, Schmaltz, PRD'02



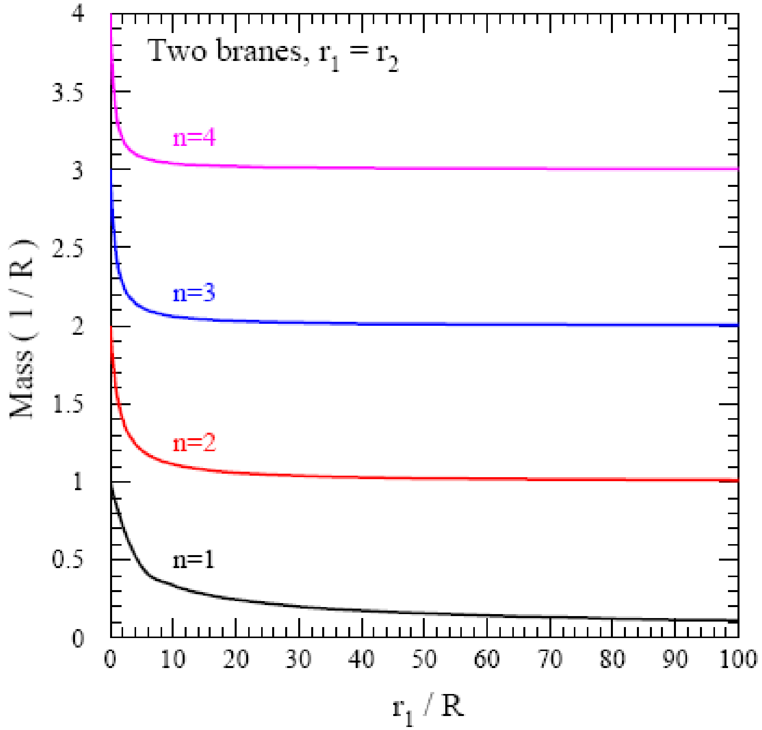
Crucial to break degeneracies in UEDs → determine cascade decays

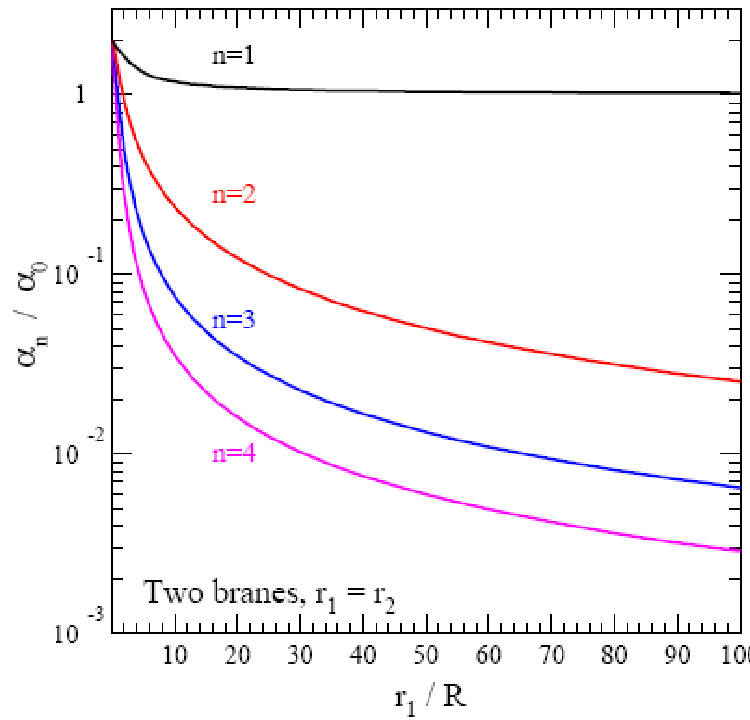
Why Brane Terms? (II)

- BKT can be very important for phenomenology

Carena, Tait, Wagner, APP'02

CTW + Delgado, Pontón, PRD 03-04, Davoudiasl, Hewett, Rizzo, PRD 03





Typically lighter modes with weaker couplings: better collider prospects

Outline

- Which brane terms?
  - Most general BKT: singularities in the thin brane limit
- How to deal with the singularities?
- Effective Description of BKT with classical renormalization
  - Renormalization Prescription: Analytic Renormalization
  - Implications of Analytic Renormalization
- UV completion: Deconstruction
  - Radiative Corrections in Deconstructioin
  - Effects of BKTs in Deconstruction
- Conclusions

Which Brane Terms?

- Most general BKT allowed by the symmetries

Aguila, Pérez-Victoria, J.S., JHEP'03

Odd-odd

$$\mathcal{L}_{\mathrm{k}} = (1+a\delta_0)\partial_\mu\phi^\dagger\partial^\mu\phi-(1+c\delta_0)|\partial_y\phi|^2+\frac{b}{2}\delta_0(\phi^\dagger\partial_y^2\phi+\mathrm{h.c.})$$

parallel

Orthogonal

Which Brane Terms?

- Most general BKT allowed by the symmetries

Aguila, Pérez-Victoria, J.S., JHEP'03

$$\mathcal{L}_{\mathrm{k}} = (1+a\delta_0)\partial_\mu\phi^\dagger\partial^\mu\phi-(1+c\delta_0)|\partial_y\phi|^2+\frac{b}{2}\delta_0(\phi^\dagger\partial_y^2\phi+\mathrm{h.c.})$$

- KK reduction

$$\left\{[1+(b+c)\delta_0]\partial_y^2+(b+c)\delta_0'\partial_y+\frac{b}{2}\delta_0''\right\}f_n = -m_n^2(1+a\delta_0)f_n$$

$$\langle f_n, f_m\rangle = \frac{1}{2\pi R}\int_{-\pi R}^{\pi R}\mathrm{d}y\,(1+a\delta_0)f_n f_m = \delta_{nm}$$

Which Brane Terms?

- KK reduction difficult to solve:
  - Regularize delta, solve numerically or analytically and take the limit of zero width
- Solution is not continuous in the parameters

	$b = 0$	$b \neq 0$
zero mode	$f_0 = \left[1 + \frac{a}{2\pi R}\right]^{-1/2}$	NO zero mode
massive modes	$f_n \propto \cos(m_n y) - \frac{a m_n}{2} \sin(m_n \lvert y \rvert)$	$f_n \propto \sin(m_n \lvert y \rvert)$
	$\boxed{\tan(m_n \pi R) + \frac{a}{2} m_n = 0}$	$\boxed{m_n = \frac{n + 1/2}{R}}$

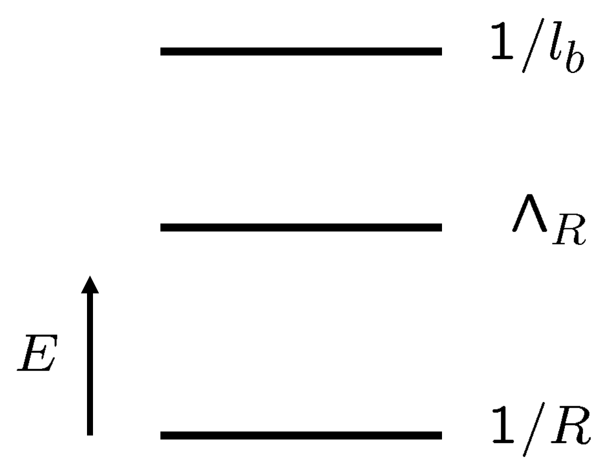
How to Deal with the Singularities?

- What do we do now?
  - Even-even orthogonal BKTs very singular: non-analytic dependence of the spectrum on their value
  - BKT (including the dangerous ones) are generated by radiative corrections
  - Gauge invariance protects gauge bosons from dangerous BKT
- Singularities can be dealt with in an effective Lagrangian approach with thin branes
  - Physics below UV cutoff smooth (analytic renormalization)
  - Physics above the UV cutoff can lead to different effective operators

Effective Description of BKT

- Models with ED: effective theories valid up to a cutoff $\Lambda = \mathrm{Min}[\Lambda_R, 1/l_b]$
- Different operators can be organized according to dimension

$$\mathcal{L} = \mathcal{L}^0 + \frac{1}{\Lambda}\mathcal{L}^{(1)} + \frac{1}{\Lambda^2}\mathcal{L}^{(2)} + \ldots$$



- Branes of any width $\leq 1/\Lambda$ give same physical results: we can take the thin brane limit
- Thin brane singularities (associated to even-even orthogonal BKTs) appear at high orders: need renormalization prescription

Analytic Renormalization of BKTs

Aguila, Pérez-Victoria, J.S., JHEP'06

- Compute order by order in the expansion in $1/\Lambda$
- Thin brane singularities $\propto \delta^n(0), \delta'^n(0), \ldots$ have to be regularized and subtracted: renormalization prescription

Analytic renormalization: set to zero all thin brane divergences at any point in the calculation.

$$\boxed{\delta^n(0) \equiv \delta'^n(0) \equiv 0}$$

- This prescription follows from an analytic regularization related to the Riemann's zeta function

$$\delta_t(x) = \frac{1}{2\pi}\left[1 + \mathrm{Li}_t(e^{ix}) + \mathrm{Li}_t(e^{-ix})\right] \xrightarrow[t \to 0]{} \delta(y)$$

Implications of Analytic Renormalization

- Even-even orthogonal BKT can be eliminated from the beginning:

$$\mathcal{L} = \bar{\psi}^- i \not{\partial} \psi^- + (1 + a\delta_0)\bar{\psi}^+ i \not{\partial} \psi^+ - \left(1 + \frac{b\delta_0}{2}\right)[\bar{\psi}^+ \partial_y \psi^- + \text{h.c.}]$$

  - Consider the field redefinition $\psi^+ \to (1 + \frac{b}{2}\delta_0)^{-1}\psi^+$

$$\mathcal{L} = \bar{\psi}^- i \not{\partial} \psi^- + \frac{(1 + a\delta_0)}{(1 + b\delta_0/2)^2}\bar{\psi}^+ i \not{\partial} \psi^+ + [\bar{\psi}^- \partial_y \psi^+ - \bar{\psi}^+ \partial_y \psi^-]$$

  - Traded orthogonal BKT with parallel times singular terms: use analytic renormalization

$$\mathcal{L} = \bar{\psi}^- i \not{\partial} \psi^- + [1 + (a - b)\delta_0)]\bar{\psi}^+ i \not{\partial} \psi^+ + [\bar{\psi}^- \partial_y \psi^+ - \bar{\psi}^+ \partial_y \psi^-]$$

Implications of Analytic Renormalization (II)

- Use analytic renormalization to eliminate as many terms as possible ...

$$
\begin{aligned}
\mathcal{L}^{(0)} &= \bar\psi(i\,\not\partial - \gamma_5\partial_5)\psi \\
\mathcal{L}^{(1)} &= \kappa_1\sigma(\partial_5\bar\psi)\partial_5\psi + a_I^R\delta_I\bar\psi_R i\,\not\partial\psi_R + a_I^L\delta_I\bar\psi_L i\,\not\partial\psi_L \\
\mathcal{L}^{(2)} &= \kappa_2\bar\psi_L\partial_5^3\psi_R + \xi_I\delta_I(\partial_5\bar\psi_L)\partial_5\psi_R + [\eta_I^L\sigma\delta_I\bar\psi_L\partial_5^2\psi_R + (L\leftrightarrow R)]
\end{aligned}
$$

- and solve order by order in powers of $1/\Lambda$

$$m_n = \frac{n}{R}\left[1 + A\frac{1}{\Lambda R} + (A^2 + Bn^2)\frac{1}{(\Lambda R)^2} + \ldots\right]$$

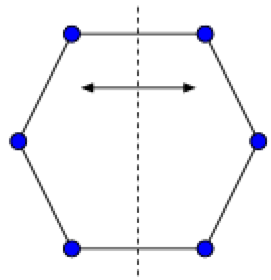
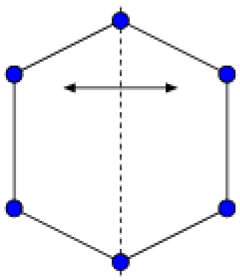
$$A = -\frac{a_0^R + a_\pi^R}{2\pi}, \qquad B = \frac{\kappa_1^2}{2} + \kappa_2$$

- Can do the same for wave functions, scalars, gauge bosons, higher orders, ...

UV Completion: Deconstruction

- 4D theory whose low energy Lagrangian is equal to that of ED

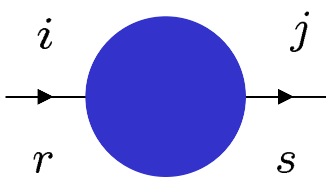
Aguila, Pérez-Victoria, J.S. '06

Vector Orbifold   Chiral Orbifold

- Link fields condense giving a discrete fifth dimension with lattice spacing $s = (gv)^{-1}$ and radius $\pi R = Ns$
- We want to investigate two aspects of these theories:
  - Generation of BKTs at one loop
  - Effects of these BKTs at low energies

Radiatively Generated BKT in Deconstruction

- Consider the deconstruction of the model with fermions and scalars studied by **Georgi, Grant, Hailu PLB'01**
- We have computed all the relevant 1PI diagrams at one loop



$$\Big|_{div} = \frac{i}{32\pi^2\epsilon}\delta^r_s\left\{\frac{\not{p}}{2}(\delta_{ij}+\gamma_5\delta_{i-j})g_\psi^2 n[4\eta\gamma_5(\delta_{i0}-\delta_{iN})+\ldots]\right\}$$

- BKT identified with different contributions at the endpoints
  - All types are generically induced
  - They are independent of N (small in the continuum limit)

Effects of BKT in Deconstruction

- Consider arbitrary BKT with coefficients $\mathcal{O}(1)$
- Expand the results in powers of the cutoff $s \sim 1/N$

$$m_n = \frac{n}{R}\left[1 + \mathcal{A}\frac{s}{R} + \left(\mathcal{A}^2 - \frac{n^2\pi^2}{24}\right)\left(\frac{s}{R}\right)^2 + \ldots\right]$$

- Reproduce the effective Lagrangian results (as it should)
- Is that all? *NO!!!*
  - Scalars with arbitrary BKTs at one brane

$$m_n = \frac{n + \frac{1}{2}}{R} + \mathcal{O}\left(\frac{s}{R}\right)$$

Localized tachyon plus massive modes

  - Can be reproduced in the effective theory with localized tachyonic masses that effectively induce (-+) bc

Effects of BKT in Deconstruction

- Fermions with mass and Wilson term not tuned at the brane:
  - Chiral Orbifold:

$$m_n = \frac{n}{R}\left[1 + \mathcal{A}\frac{s}{R} + \left(\mathcal{A}^2 - \frac{n^2\pi^2}{24}\right)\left(\frac{s}{R}\right)^2 + \ldots\right]$$

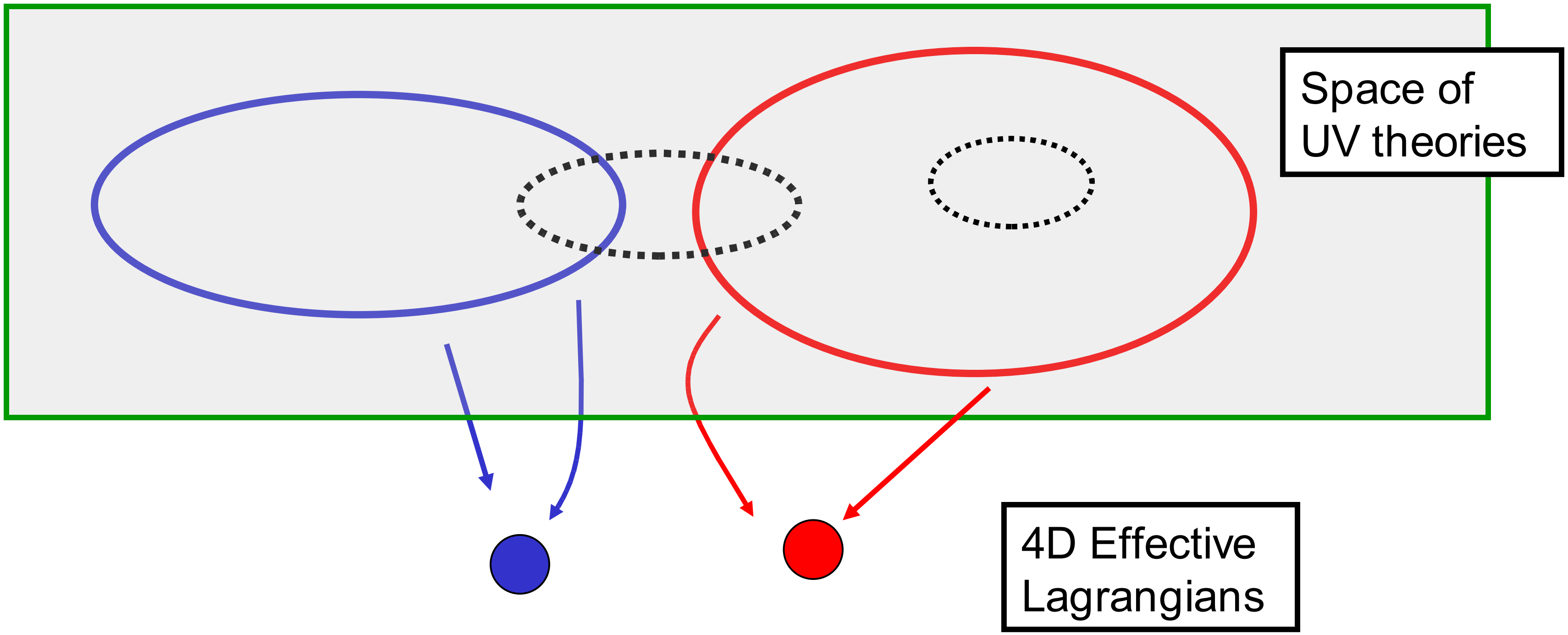
  - Vector-like Orbifold:

$$m_n = \frac{n+\frac{1}{2}}{R}\left[1 + \mathcal{B}\frac{s}{R} + \left(\mathcal{B}^2 - \frac{(n+\frac{1}{2})^2\pi^2}{24}\right)\left(\frac{s}{R}\right)^2 + \ldots\right]$$

- Can be reproduced in the effective theory with mass mixing with brane fermions

Effects of BKT in Deconstruction

- Different UV completions can be classified in universality classes (giving the same low energy effective Lagrangian)



- Low energy physics can be described with the appropriate effective Lagrangian

Conclusions

- Brane Kinetic Terms are always present and can have important phenomenological implications
- Some BKT give raise to singularities in the spectrum
- An effective Lagrangian approach with delta-like branes, supplemented with (classical) analytical renormalization gives simple, smooth, sensible physical results
- UV completions can belong to different universality classes: different low energy effective Lagrangians
- Phenomenological applications with small, perturbative, BKTs are well motivated. Large BKTs require UV assumptions that are not necessarily protected by symmetries